Raise, raise the Exponent High!

Math Songs to Familiar Melodies

An after dinner sing-along led by John Kiltinen

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Several Mathematical Homages
words © 2006 by John O. Kiltinen
(Tune: “O Tannenbaum” (O Christmas Tree))

An Homage to the Marquis de l’Hospital

O l’Hospital, O l’Hospital, the rule is yours, you bought it,
O l’Hospital, O l’Hospital, but John Bernoulli thought it,
Some think you were a noble wuss, to buy another’s calculus,
O l’Hospital, O l’Hospital, the rule is yours, you bought it.

An Homage to Richard Dedekind

O Dedekind, O Dedekind, your cuts are oh so clever,
O Dedekind, O Dedekind, your cuts are oh so clever,
And using these on rationals to build the reals is such a thrill,
O Dedekind, O Dedekind, your cuts are oh so clever.

An Homage to Karl Weierstrass

O Weierstrass, O Weierstrass, your epsilons and deltas,
O Weierstrass, O Weierstrass, your epsilons and deltas,
They make a limit very clear, and also not so far from here¹,
O Weierstrass, O Weierstrass, your epsilons and deltas.

¹ Explain “here” by pointing to your head.

An Homage to William Rowan Hamilton

O Hamilton, O Hamilton, your three square roots of minus one,
You carved them on the bridge, at least that’s what you wrote to your dear son.
I think that they are very cute, though I regret they don’t commute,
O Hamilton, O Hamilton, they generate quaternions.

An Homage to Leopold Kronecker

O Kronecker, O Kronecker, you said God made whole numbers,
O Kronecker, O Kronecker, you said God made whole numbers,
And that the rest was work of man, best suited for the garbage can,
O Kronecker, O Kronecker, you said God made whole numbers.

An Homage to Carl Friedrich Gauss

O Carl Gauss, O Carl Gauss, you with straight-edge and compass,
O Carl Gauss, O Carl Gauss, you with straight-edge and compass,
Made polygons sides seventeen, this matched your age, you were a teen,
And then you went beyond this feat to study integers complex.
The Continuous Adding Song (That’s all integration is)
words © 2005 by John O. Kiltinen
(Tune: “Walk right in, sit right down”)

1. Not just by twos, threes, fours or fives,
   We can add continu’sly.
   Not just by twos, threes, fours or fives,
   We can add continu’sly.
   Everybody’s gladdin’ bout a new way of addin’,
   Don’t it truly blow your mind.
   Not just by twos, threes, fours or fives,
   We can add continu’sly.

2. Stretch that S, S is for sum,
   It becomes an integral sign.
Stretch that S, S is for sum,
It becomes an integral sign.
Everybody’s gladdin’ bout a new way of addin’,
That’s all integration is.
Stretch that S, S is for sum,
It becomes an integral sign.

3. Bridge the gap, from old to new,
   Finite to continuous.
Go the limit, sneak up on it,
That’s the role of Riemann sums.
Everybody’s gladdin’ bout a new way of addin’,
With a limit sneak up on it.
Bridge the gap, from old to new,
That’s the role of Riemann sums.

4. If this seems just too hard to grasp,
   Ponder your odometer.
   It adds up the miles that you drive,
   Doing so continuously.
   Everybody’s gladdin’ bout a new way of addin’,
   Meters do it all the time.
   If this seems just too hard to grasp,
   Then ponder your odometer.

5. F of x, then times a dx,
   They’re the things that we add up.
The dx is so very thin,
And x flows from left to right.
Everybody’s gladdin’ bout a new way of addin’,
Areas of little thin strips.
F of x, then times a dx,
They’re the things that we add up.

6. Now we have a new way to add,
   Aren’t we rather proud of us.
   No more two, three, four at a time,
   We just let the summands flow.
   Everybody’s gladdin’ bout a new way of addin’,
   They flow in and we add them up.
No more two, three, four at a time,
We just let the summands flow.

(Reprise and finale. Repeat.)

No more two, three, four at a time,
Baby let those summands flow on.
Calc Test Thursday Morning  
words © 2006 by John O. Kiltinen  
(Tune: “I’m getting married in the morning,” lyrics by Alan Jay Lerner and music by Frederick Loewe, from “My Fair Lady.”)

1. I’ve got a calc test in the morning,  
Whir, whir, the gears are going to whine,  
Got this stuff mastered, so let’s go get plastered,  
Just get me to the test on time.  
I’ve got a calc test in the morning,  
So tough it messes with your mind,  
Before it’s all over, I know I’ll be sober,  
Just get me to the test on time.  
If I’m still drinking just one beer more  
And then I’m thinking whisk me out the door  
For I’ve got a calc test in the morning,  
I think I know this stuff all cold,  
So eat, drink, be merry, it won’t be so scary,  
Sobriety can wait ’til I am old.

2. I’ve got a calc test in three hours,  
Head aches, I think it will explode,  
Still far from sober, I’m awfully hung over,  
Oh pity me, I bear a heavy load.  
I’ve got a calc test in three hours,  
Clunk, clunk, the gears are going to grind,  
Poor brain will stall out, alc’holic fallout,  
Is not too helpful for the thinking mind.  
If I am puking, just one wretch more,  
Then I am thinking peel me off the floor,

For I’ve got a calc test in three hours,  
Must rid my throat of sour slime,  
No shape to take it, will just have to fake it,  
But get me to the test on time.

3. I flunked a calc test Thursday morning,  
Was overconfident it seems,  
The next will be different, from boozing I repent,  
I’ll study ’til it permeates my dreams.  
I flunked a calc test Thursday morning,  
So bad it broke my mother’s heart,  
But had to tell her, knew she would beller,  
Cuz they are paying for a kid who’s smart.  
If I am thinking I’m so prepared,  
I’ve time for drinking I am full of merde.  
I vow next time I’ll do much better,  
I’ll study hard and walk the line,  
Good night of snoozing, to hell with the boozing,  
Just get me to the test on time.

Reprise:  
Good night of snoozing, to hell with the boozing,  
Just get me to the test, get me to the test,  
For gawd’s sake get me to the test on time.
The Quadratic Formula Song
words © 2006 by John O. Kiltinen
(Tune: “Ta-Ra-Ra-Boom-Der-É,” 1891, by Henry J. Sayers)

1. If a function you have found,
   Climbs up smartly off the ground,
   Not a line, it’s plain to see,
   Slope is growing linearly.
   First the graph is going down,
   Then it changes all around,
   Graphs as a parabola,
   To its roots you say ¡hola!

Refrain:
Quadratic formula, quadratic formula,
My heart you warmula, quadratic formula.
Quadratic formula, quadratic formula,
My heart you warmula, quadratic formula.

2. If the highest power is two,
   This is all you have to do,
   When you want to find the roots,
   Here’s the plan, now listen Toots.
   I will tell you only once,
   ’Cause I know you’re not a dunce,
   You can do this, wait and see,
   It’s as easy as can be.

Refrain:

3. Take the coefficients three,
   Name them a and b and c,
   a goes with the power of two,
   b’s with x, you know it’s true,
   c’s the one that’s all alone,
   Causing it to moan and groan,
   Now they’re all identified,
   Roots, no longer can you hide.

Refrain:

4. Start with minus b upstairs,
   Then because the roots are pairs,
   First you add and then subtract
   The next part, that is a fact.
   This next part is a square root,
   I just know you give a hoot,
   Here’s the root, now listen please,
   b squared minus four a c’s.

Refrain:

5. Now we’ll do the lower part,
   You’ll soon know it all by heart,
   All we do is double a,
   Just because we feel that way.
   Put it all below the bar,
   Now, we’re done, so there you are,
   We have given both the roots,
   Exposed them in their birthday suits.

Refrain:

6. Now you know just what to do,
   If the roots you need to view,
   Of a polynomial,
   With a parabolic feel,
   Use this magic formula,
   To the roots you’ll say ¡hola!
   It will work if they are real
   Or complex, it’s such a deal.

Refrain:
The Trigonometry Song
words © 2006 by John O. Kiltinen
(Tune: “Um Ya Ya, Um Ya Ya”, the St. Olaf College fight song)

Refrain:
Sine, cosine and tangent, cotangent
and secant,
Cosecant, remember with soh-cah-toa.
Sine, cosine and tangent, cotangent
and secant,
Cosecant, remember with soh-cah-toa.

1. Our trig class is coolest although we act foolish,
We sometimes get giddy and sing silly songs.
We learn all these functions and lose our compunctions,
’Bout making mistakes ’cause we fix all our wrongs.

Refrain:

2. Our trig functions start out with triangles laid out
But soon they are going in circles you see,
The concept starts simple but like a small pimple
It soon grows much riper and fester on me.

Refrain:

3. The first three are heav’n sent sine cosine and tangent
They’re what you will see almost all of the time.
But sometimes you’ve need of one over the leader,
That’s just when the others will work out so fine.

Refrain:

4. Let’s start with the sine graph, it follows a fine path,
It rises and falls very regularly.
From high point of one it to minus one plummets
Its period we find is just two pi you see.

Refrain:

5. Once you have the sine cold the cosine will unfold,
The graph of the cosine is almost the same.
Just push the sine leftward by pi over two, Pferd
And you have a cosine graph that is its name.

Refrain:

6. The tangent is diff’rent its graph is a tall gent,
It grows without bound to the upside and down.
This tall gent has clones, Merv, that mimic his one curve
A new one is born every pi all around.

Refrain:

7. Now if you’ve a need, pal, to graph a recip’cal,
Of one of these basic ones here’s all you do.
Just make all the short tall and tall short, that is all
Infinity, zero, they switch places too.

Refrain:
8. These trig functions diverse, both forward and inverse
Are subject to many cool identities.
They come by the myriad, you’ll go through a period
Of proving them true so your teacher you’ll please.

Refrain:

9. When time comes to invert a function we revert,
To graphing the one we would like to reverse.
We use the graph backwards as if it were forwards,
And relegate all but one branch to the hearse.

Refrain:

10. And if it seems absurd, this using graphs backward,
The answer we find is just give it a flip.
Just flip the graph over the diag’nal brother,
Then graph again forward if that is your trip.

Refrain:

High Math Hopes
words © 2006 by John O. Kiltinen
(Tune: “High Hopes”)

When all is slow and seems nowhere to go,
Just push on, wee little steps too can grow.

What makes that harmonic series,
Think it’ll climb as high as my knees,
Anyone knows that wee stuff,
Can’t add up fast enough,

But it has high hopes, it has high hopes,
It has high apple pie, in the sky hopes,

So if your progress seems too slow,
Keep on adding, Bro,
Think harmonic series,
Oops there it grows up past the tops of the trees.

When thunder rolls, and your proof’s full of holes,
That’s no reason to quit, or say t’ell with it!

Just what made that dude Andrew Wiles,
Think his quest would end in broad smiles,
That he could prove what Fermat saw,
And repair his own flaw,

But he had high hopes, he had high hopes,
He had high apple pie, in the sky hopes,

So even if ten thousand score,
Tried and failed before,
Failed to prove old Fermat,
Oops, unproven theorem’s now a firm law.

Conjectures that have been around,
Drove lots to the ground,
Solvers thunk and they thunk,
Oops there goes a Hilbert problem kerplunk.
1. Blessed “don’t care-iance” things that don’t change,
When their creators traverse o’er their range.
These are invariants, anchoring points,
While all around them is like out of joint.

Refrain:
This is my glory, ’cause I’m a math geek,
Each new invariant’s the thrill of the week.
This is my glory, ’cause I’m a math geek,
Each new invariant’s the thrill of the week.

2. Start with quadratics of degree two,
All graph as conics, but who’s of them who?
Take the discriminant, look at its sign,
Ellipse or the others, it tells every time.

Refrain

3. Don’t stop at degree two,
discriminant’s free,
For all polynomials of every degree.
And it’s invariant, don’t ask me just how,
If only I knew, I would tell you right now.

Refrain

4. Think of dimension of a neat vector space,
It’s as invariant as the nose on your face,
You pick a basis, and I’ll do the same,
They’ll be the same size, dimension’s its name.

Refrain

5. Take the determ’nant of a matrix that’s square,
It’s an invariant, I know that you care.
If you go sim’lar, to one that you’ve named,
You will discover, determ’nant’s unchanged.

Refrain

6. The determinant’s invariant under adding of rows,
And that is not all, not as far as this goes.
Swap any two rows and you surely will find,
You’ve changed the determ’nant, but only its sign.

Refrain

7. Parity too’s an invariant of sorts,
Even or odd, each inquirer reports,
One might get six, and another get four,
But both get even, an invariant score.

Refrain

8. Parity’s also unsubject to change,
When conjugation is given free range.
Take from a scramble a conjugate one,
Get the same thing as you got on first run.

Refrain
Onward, Three-D Vectors
words © 2006 by John O. Kiltinen
(Tune: “Onward, Christian Soldiers”)

1. Onward three-D vectors,
   Little pointy spears,
   Making tall cross products,
   With divergent peers.
   Each its own direction,
   Its identity,
   That and length determine,
   What vector it be.

Refrain:
   Onward three-D vectors,
   Little pointy spears,
   Making tall cross products,
   With divergent peers.

2. What is the direction,
   This cross product makes?
   From the plane its parents form,
   Stands erect in space.
   And it gets its magnitude,
   From its parents two,
   And the angle that they form,
   It enters in there too.

Refrain

3. But we need to make a choice,
   From two we must select,
   For that fine cross product,
   To their plane erect.
   Could go either up or down,
   Here’s the choosing tool.
   When you need the aim of it,
   Just use the right hand rule.

Refrain

4. If two vectors seem to form,
   A right angle true,
   But you want to be quite sure,
   This is all you do.
   Figure their dot product,
   Scalar that it be,
   If you get a zero dot,
   Then you are home for free.

Refrain

5. Vectors also can be summed,
   \( \vec{A} \) plus \( \vec{B} \) yields \( \vec{C} \),
   Method’s geometrical,
   Easy as can be.
   Slide the tail of vector \( \vec{B} \),
   To the point of \( \vec{A} \),
   Go tail of \( \vec{A} \) to point of \( \vec{B} \)
   And get \( \vec{C} \) just that way.

Refrain

6. Like a vector army,
   Taking to the field,
   Many pointy arrows,
   Best deploy your shield.
   Lest you get impaled by them,
   And in death you fall,
   For a well-formed vector field,
   Will make a spiky ball.

Refrain
My Favorite Rings
original words and concept by Karen Aucoin,
modified by John Kiltinen, copyright © 2006
(Tune: “My favorite things” from “The Sound of Music”)

1. Integers, all types, plus, zero, and minus,
   Matrices square, equal width to their highness,
   Integers modulo all finite things,
   These are a few of my favorite rings.

   Add to a base ring a number of unknowns,
   X one, two, three, of each other they are clones,
   Add them and multiply, it makes me sing,
   Lengthens the list of my favorite rings.

Refrain:
When I’m asked to prove a theorem, And I’m feeling blue,
I simply remember my favorite rings, And then I know what to do!

2. Take any ring, on it take all the functions,
   Add them and multiply, with no compunctions,
   Do the ops pointwise, it is just the thing,
   Now I have build a new favorite ring.

   Rings do not have to be like this, quixotic,
   Just take a number set far less exotic
   Rationals, reals, or more complex things,
   Put on my list of my favorite rings,

Refrain:
On a down day I just sort them, These commute, these don’t,
I sort and I pile up my favorite rings, And then staying blue, I won’t!

Several Mathematical Toasts
words © 2006 by John O. Kiltinen
(Tune: “Minnesota Rouser”)

A toast to Pythagoras
Lift your glass to Pythagoras, For his theorem warrants a little fuss,
Right triangle’s hypotenuse Has a square that is the same
As the squares summed, strong and lame, Of the two legs of it.

A Toast to Évariste Galois
Lift your glass to Évariste Galois, Teenage genius, vive le roi!
Hail the king of algebra he, Solved big problems, got no fee
Got a theory named pour lui, And fought a fatal duel.
Counting Roots of Unity
words © 2006 by John O. Kiltinen
(Tune: “I count my blessings instead of sheep” by Irving Berlin.)

When I’m worried and I can’t sleep,
I count roots of unity instead of sheep,
And I fall asleep counting roots of unity.

When roots complex they start to spew,
I think when real ones were all I knew,
And I fall asleep counting the complex roots.

I think about $n$th roots on a circle even spaced,
And single out the primitive to see where they are placed.

So if you’re worried and you can’t sleep,
Count roots of unity instead of sheep,
And you’ll fall asleep counting roots of unity.

Transfinite Sheep
words © 2006 by John O. Kiltinen
(Tune: “Brahms’ Lullaby.”)

1. Go to sleep, my math geek,
   Counting transfinite sheep.
   First you count the finite ones,
   Then omega when done.
   Next omega plus one,
   Et cet’ra my son,
   Keep it up, ’til you’re through,
   Then omega times two.

2. Go to sleep, my math geek,
   Counting transfinite sheep,
   We were at two omega,
   Keep on going without flaw,
   Add one more to go on.
   And go on, on, and on.
   Keep it up, glory be,
   Then omega times three.

3. Now that you, know these rules,
   Time to give you new tools,
   When you’ve done two, three, four, five,
   Times omega sakes alive.
   Cap omega times $k$,
   For each natural $k$,
   Listen now, be prepared,
   It is omega squared.

4. I think now you can see,
   What the next have to be,
   Take the powers, finite ones,
   Of omega my dear son,
   In a while you will get,
   To a transfinite state.
   If you aren’t yet asleep,
   Let’s quit counting these sheep.
\[ e^{i\pi} = -1 \]

words © 2006 by John O. Kiltinen
(Tune: Notre Dame University fight song)

Raise, raise the exponent high,
e is the base, the power is \( i \),
Times a pi that’s up there too,
No need to square, that’s all we do.
What do we get, you’d think it’s a mess,
But you’d be wrong if that is your guess,
For the answer’s minus one,
Go ponder its meaning, son.

You’ll Never Code Alone
words © 2006 by John O. Kiltinen
(Tune: “You’ll Never Walk Alone”)

When you code up a storm keep your standards high,
And don’t be afraid of the bugs,
When your code monkey’s there, perched on shoulder or thigh,
Giving signals with nudges or slugs
He’ll keep you on track,
He has such a knack,
Catching missing this’s or that’s
Code on, code on with hope in your heart,
And you’ll never code alone,
You’ll never code alone.

Hail to the Fundamental Theorem of Calculus
words © 2006 by John O. Kiltinen
(Tune: University of Michigan Fight Song)

Hail to the Fundamental
Theorem of Calculus,
Hail to the guys who found it,
We think they’re swell.
It led to modern science,
It makes our life much better,
Hail, hail the F. T. C.
We vow to use it well!
Those Were the (Math/CS) Days
words © 2006 by John O. Kiltinen
(Tune: “ Those Were the Days,” from “All in the Family.”)

1. (Math version)

Boy those double E’s were cool,
Squaring things on their slide rule,
Our only calculation tool,
Those were the days!

Slide rule grease was Vaseline,
Add on paper for a mean,
Mister we could use some mental
math dudes back on the scene.

Didn’t use no laptops yet,
Books of tables were a hit,
Web was what a spider set,
Those were the days!

2. (Computer science version)

Those old coders had a knack,
For programming the ENIAC,
Didn’t even need to hack,
Those were the days!

And you knew who you were then,
Wore a tie for IBM,
Mister we could use some well-
dressed, crew-cut coders again.

Didn’t need no gigabytes,
Just a thousand blinking lights,
Punch cards boxes to the heights,
Those were the days!

Some Rounds
words © 2006 by John O. Kiltinen
(Tune: “Frère Jacques”)

Partial fractions, partial fractions,
Or by parts, or by parts,
Integration methods known among the
best heads,
In the world, in the world.

Algorithms, algorithms,
Basic rules, not for fools,
Methods that we follow, in them we
can wallow,
Our math tools, our math tools.

Some Rounds
words © 2006 by John O. Kiltinen
(Tune: “Row, row, row your boat”)

Keep, keep, keep yourself,
Off th’expanders’ team,
Factoring’s better most always it’s
better,
Expand in the extreme.

Don’t, don’t, don’t forget,
When you multiply,
Zero times anything, yes I mean
anything,
Goose egg in reply.
I’m My Own Barber  
words © 2006 by John O. Kiltinen  
(Tune: “I’m My Own Grandpa”)  

It sounds funny I know, but it really is  
so, oh I’m my own barber.  

I’m my own barber.  I’m my own  
barber.  It sounds funny I know, but  
it really is so, oh I’m my own  
barber.  

1.  
Now many, many years ago when I  
was very young,  
And I had hair.  Oh yes I did, it to my  
shoulders hung.  
I lived back then in the U.P., in a small  
but comfy town.  
And I cut hair, that was my job, for  
guys from all around.  

2.  
I shaved them too, cut whiskers off of  
each and every chin,  
Of every guy who to my shop would  
happen to come in,  
I got quite good, my fame it spread  
down to the county seat,  
“Don’t shave yourself,” the guys  
would say, instead go visit Pete.  

3.  
Yes Pete’s my name, it said just that  
upon my barber’s sign,  
The guys would come, I’d clip them  
clean, and they’d go down the mine,  
And more would come, they liked my  
work, it was the best you see,  
My shaves were clean, my haircuts  
neat, as neat as they could be.  

4.  
I got too busy from my fame and  
thought I should cut back,  

So I could spend the morning hours  
just lounging in the sack.  
I changed my sign, wrote words up  
there that really blew my valves,  
“I’ll shave those guys–but just those  
guys–who do not shave them-  
selves.”  

5.  
It seemed to work, they understood,  
they did not piss and moan,  
When I would see a razor burn and  
send a shaver home.  
It worked quite fine for just a week, I  
got my morning rest,  
But then a kid, a high school kid,  
came in with a request.  

6.  
He said he had a question that just  
needed to be asked,  
I calmly said, “Just fire away!” I was  
soon to be unmasked.  
So ask he did, and when he did, it  
shook me through and through,  
For that young whippersnapper asked,  
“And who the heck shaves you?”  

7.  
My lightning mind soon grasped the  
flaw this snot-nosed kid had found,  
Though he just stumbled on it, once  
it’s asked makes heads spin ’round,  
For if I shave myself, I make a lie out  
of my sign,  
And if I don’t I’m lying too, I’m really  
in a bind.  

8.  
For ’bout a month I pondered it, and  
grew a shaggy beard,
It was a real paradox, much worse than I had feared.
To fix the problem for all time, there was but just one way,
To tuck my tail between my legs and slyly steal away.

9.
So if you ever find, my friend, that you are in my place,

The Derivative Rules Song
words © 2006 by John O. Kiltinen
(Tune: “Twinkle, twinkle little star”)

1. When I learned derivatives
   Thought I’d die, but yet I lived,
   Coolest stuff you’d care to find,
   Tells you what’s the rate of change,
   Of a function that’s in range.

2. First the task they threw at me,
   Give the prime of f plus g.
   It’s so easy, really, chum,
   Sum the primes to prime the sum,
   It gets harder, wait and see.

3. Next the task they throw at me,
   Find the prime of f times g.
   Start with f times g prime, Twit,
   Add an f prime g to it,
   Times a g prime, ’round the bend.

4. Quotient Rule is next, my friend,
   Seems these rules will never end,
   g prime f less f prime g,
   Over f squared, don’t you see,
   That the quotient rule is neat,
   If you don’t admit defeat.

5. Chain Rule’s next, hang in there friend,
   We are getting to the end.
   (f of g) prime that’s our goal,
   (f prime of g) it takes the role,
   Of the factor first, my friend,
   Times a g prime, ’round the bend.

6. Those are all the gen’ral rules,
   They are all the basic tools,
   Others are more heaven-sent,
   Like the rule for exponents,
   And the trig and log rules, pal,
   Learn them and you’ll never fail.